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One feature in the assessment of investments in a recession

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Abstract:

The availability of free funds in the economy is usually associated with the investment process. In the specialized literature, various methods for assessing investments are known, but one of the fundamental methods of assessing investments is related to the calculation of the NPV (Net Present Value) indicator. According to the generally accepted rule, an investment is profitable if there is an NPV greater than zero. A negative NPV indicates that the investment is loss-making and should be rejected. However, this rule is derived under certain conditions. This paper shows a peculiarity in the assessment of investments related to the fact that in conditions of recession there are situations in which an investment with a negative NPV can be accepted. The article uses financial mathematics methods, in particular formulas for accruing interest and discounting future cash flows. Three options are considered in the presence of free cash – to keep it at home, to deposit it on a bank and to invest. Through concrete examples, it has been shown that in conditions of low deposit interest rates and high bank account service fees, it is possible that an investment with a negative NPV would be preferable, which refutes the generally accepted claim in theory.

Keywords:

Investment; NPV.

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Introduction

In the presence of free financial resources, practice shows that there are three options:

1) We can keep the money at home (*first case*).

2) We can deposit the money in a bank (*second case*).

3) We can make investments (*third case*).

At the first opportunity, they do not change their value, are not in a safe place and depreciate.

With the second option, given the extremely low interest rates on deposits in recent years and the increasing amount of service charges, with a low amount of the deposited amount, on the one hand, we believe that it is safe and will be preserved, but on the other hand, it may become less than the amount originally deposited over time.

The third option, which we would say is the riskiest, but with it the return is usually significant and this makes it the most preferred in recent times. This option is associated with an assessment of the respective investment. In the evaluation of investments, the concept of *NPV* (net present value) is known (Bodi, Keyn & Markas, 2000; Galits, 1994; Dochev, Nikolaev & Petkov, 2010; Nikolaev, Suruzhon, Stoyanov, Zapryanova, Milkova & Miryanov, 2021), which we will talk about in more detail in the following exposition. Here we would like to note that *NPV* serves as an indicator whether or not it is worth making the investment. The rule applies that if $NPV \geq 0$ the investment can be accepted, and if $NPV < 0$, the investment should be rejected.

In our opinion, in contemporary conditions, this rule is not always valid. It is imperative to take into account the fact that in times of recession there are low interest rates on deposits in banks, a high inflation rate and high fees for servicing accounts with banks.

It turns out that this method is applicable under certain conditions. If we add new conditions to the model that correspond to a specific economic environment, other results can be obtained. Now we take into account the fact that interest rates on deposits in banks are very low, while at the same time there are relatively high monthly fees for servicing an account with the bank. All this affects the validity of the theoretically derived formulations, which do not take into account the above factors.

The aim of this article is to prove that, under certain conditions, it is valuable to invest in a project with a negative *NPV*.

The goal is achieved by performing two main tasks:

1) The essence of compound discounting and the method of valuing investments at a net present value are theoretically presented.

2) Tasks with specific sample data were constructed where it appears that there are situations of profitable investments, even with negative net present value.

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Theoretical basics of compound discounting and valuation of investments

When considering the time value of money, two main approaches are applied – remuneration and discounting. In practice, it makes sense to apply compound interest and a compound discount accordingly. Through remuneration, the future value of some amount of money is determined.

In case of compound remuneration, the following formula is used:

$$K_n = K\left(1 + \frac{p}{100}\right)^n.$$

It determined after the *n* period of remuneration, at *p*% compound annual interest for one period, to what amount K_n increases the amount of *K* BGN.

In general, four parameters are involved in the compound interest formula, and four types of task can arise on this basis or three of the parameters are known and the fourth is sought. For example, you can search for the interest rate by known original amount, number of periods of remuneration and increased amount. Interesting is the case in which the number of periods of remuneration, the interest rate and the increased amount are known here and the value of the original amount is sought.

If the amount due in the future in the amount of K_n BGN is known, and we need to determine its present value, which is denoted by K_E BGN, it will be:

$$K_E = \frac{K_n}{\left(1 + \frac{p}{100}\right)^n}.$$

Formula (1) is used in calculating a mathematically compound discount. If necessary, a conformal interest rate is used. A conformal interest rate is used when complex interest is applied, the annual compound interest rate is known and it is necessary to set an interest rate for a smaller period of time, for example, month, quarter and others.

For example, if the amount of BGN 2210.58 is due after four years at 0.12% compound annual interest, then determine the value of the discounted amount at the present time. We have to use formula (1). We know that K_n=2210.58 BGN, *p*=0.12% per year, the periods are years and their number is four, i.e. *n*=4. Therefore:

$$K_E = \frac{2210.58}{\left(1 + \frac{0.12}{100}\right)^4} = 2200\ BGN.$$

On this theoretical basis, we proceed to the evaluation of investments.

Investing capital in an asset for profit over time is called an investment. For example, if we do not have our own home, but live in an apartment and buy one to live in it, this is not an investment. If we buy a home in order to sell it in the future at a higher price, this is an investment.

When an investment is made, it is usually long-term, for a period of 2, 3, 5 and more years.

Let's go back to formula (1) for complex discounting. It makes it possible to calculate the present value *K* of a future amount K_n after *n* periods at an accumulation rate ahead of time for one period *p*%.

For example, let there be a proposal to invest an amount of BGN 100,000 as the amount to be received in 4 years is BGN 120,000.

According to formula (1) of the current value of the investment is BGN 100,000 and the future value is BGN 120,000, the periods are $n = 4$ and formula (1) obtains:

$$100{,}000 = \frac{120{,}000}{\left(1+\frac{p}{100}\right)^4} \quad \Rightarrow \quad 1+\frac{p}{100} = \sqrt[4]{\frac{120{,}000}{100{,}000}} \quad \Rightarrow$$

$$\Rightarrow \quad p = \left(\sqrt[4]{\frac{120{,}000}{100{,}000}} - 1\right).100 \approx 4.66\%$$

Therefore, the average annual percentage rate of profit is 4.66%.

Now we will consider a case where, after investing at a given moment, future incomes are multiple and not in the form of one-time income.

An amount of BGN 150,000 has been invested with an assumed average annual profit of 4%, as the future revenues are as follows: after the first year BGN 60,000; after the second year BGN 60,000; after the third year BGN 40,000; after the fourth year 20000 BGN

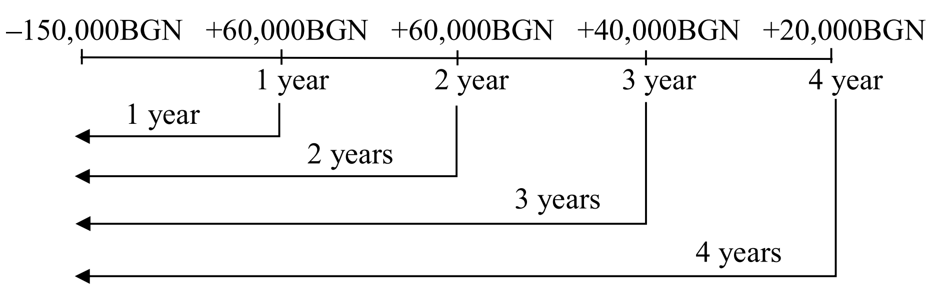
(a) Find the present value of all future cash flow from revenue.

(b) Compare the cost of the investment of BGN 150,000 and the present value of future cash flow.

c) Analyse whether the investment is profitable.

The entire financial operation is illustrated in Figure 1.

Figure 1. Graphic illustration on financial operation



Source: Created by the authors

(a)The present value of the future cash flow will be obtained by summing the discounted values of future incomes according to the time of their income, i.e.:

$$\frac{60{,}000}{\left(1+\frac{4}{100}\right)} + \frac{60{,}000}{\left(1+\frac{4}{100}\right)^2} + \frac{40{,}000}{\left(1+\frac{4}{100}\right)^3} + \frac{20{,}000}{\left(1+\frac{4}{100}\right)^4} = 165{,}821.62\ BGN.$$

b) The present value of future incomes is BGN 15,821.62 greater than the cost of the investment. This means that the real current cost of the investment is BGN 165,821.62, and what is required as an investment at the moment is BGN 150,000, i.e. the price is understated. This conclusion allows an answer to be given to point (c).

(c) Based on the results of point (b), it follows that the investment is profitable/valuable.

Let us summarize theoretically the condition and the results in the last example. We introduce the following indications:

I — cost of investment;

n – number of periods during which the investment brings incomes;

F_i – future value of income at the end of the i-th period (i =1,2, ..., n);

r – average annual percentage of profit;

PV – present value of all future cash incomes flow.

Then

$$PV = \frac{F_1}{1 + \frac{r}{100}} + \frac{F_2}{\left(1 + \frac{r}{100}\right)^2} + \cdots + \frac{F_n}{\left(1 + \frac{r}{100}\right)^n} = \sum_{i=1}^{n} \frac{F_i}{\left(1 + \frac{r}{100}\right)^i}.$$

The previous example demonstrated that if $PV > I$, then the investment is profitable. The difference between PV and I is usually denoted by NPV and is called pure present value.

From the reasoning made, we come to the following conclusion:

If

$$NPV \geq 0$$

the investment is profitable and it is advisable to invest in it.

If

$$NPV < 0$$

The investment is not profitable and it is not advisable to invest in it.

We must make the following clarifications.

If two or more investments need to be compared, then this can be done on the basis of comparing NPV in the case where for all of them I is the same and r is the same

$$NPV_i > NPV_j,$$

it is preferable to invest in the i-th investment to the j-th investment.

Often r is understood as the required average annual percentage of profit by the investor.

This means that when an investor makes an analysis of different investments, for him r is the same, and if the investment prices are different, he cannot arrange his preferences according to their NPV. In such cases, additional analyses are required, through other approaches, which we will not consider here.

An example of a valuable investment with a negative net present value

Let the initial amount to invest is K and the investment horizon is after n months. At the same time, with a fixed-term monthly deposit, the monthly interest rate

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should be p%. We will keep in mind that banks usually charge a monthly fee t to service the deposit.

We will look at the three options we pointed out at the beginning. Based on the entered markings, we will derive the value of the future amount, provided that the owner of free cash chooses each of these three options.

In the **first option** it is clear that the amount K remains the same after n months.

In the **second option**, we will first see what is the amount after n months, if there are no fees. With term deposits, remuneration is complex and the formula for obtaining the increased amount after n months is known, namely:

$$K_n = K\left(1 + \frac{p}{100}\right)^n$$

or if we lay

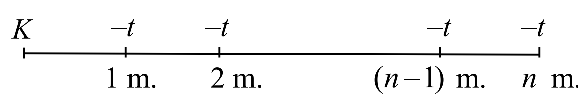
$$q = 1 + \frac{p}{100}$$

follows

$$K_n = Kq^n \tag{1}$$

Let us now consider the case if a fee of t BGN is deducted from the account on a monthly basis (Figure 2).

Figure 2. Graphic illustration on financial operation



Source: Created by the authors

At the end of the first month, the amount had risen to Kq.

At the end of the second month, after remuneration, the amount is:

$$(Kq - t)q = Kq^2 - tq.$$

At the end of the third month, after remuneration, the amount is:

$$(Kq^2 - tq - t)q = Kq^3 - tq^2 - tq \quad \text{etc.}$$

At the end of the n^{th} month, after remuneration, the amount is:

$$Kq^n - tq^{n-1} - tq^{n-2} - \cdots - tq$$

and if the last fee for K_n is deducted:

$$K_n = Kq^n - tq^{n-1} - tq^{n-2} - \cdots - tq - t$$

or

$$K_n = Kq^n - t(q^{n-1} + q^{n-2} + \cdots + q + 1).$$

In the parentheses, in the diminutive, we have a sum of the members of a geometric progression of n terms, with a first term 1 and a private q and it is equal to:

$$\frac{q^n - 1}{q - 1}.$$

Then for K_n we obtain:

$$K_n = Kq^n - t.\frac{q^n - 1}{q - 1}. \tag{2}$$

Let us now consider the **third option** of action in the presence of free financial means. First, we will clarify some concepts. The amount K to invest is assumed to be called the cost of the investment and is denoted by P_0. On the other hand, at certain periods of time until the n[th] month, a whole cash flow of different amounts is formed, which we will denote by $F_1, F_2, ..., F_n$, specifying that some of them may be negative or zero.

In general, if $F_i = 0$ for some $i = 1, 2, ..., n$, then this means that in the i^{th} month there is no expense or income for the investor, and if $F_i < 0$ for some $i = 1, 2, ..., n$, then this means that the investor, in addition to the initial expense P_0, in the i^{th} month makes an expense of F_i. If $F_i > 0$ for any $i = 1, 2, ..., n$, then for the investor there is income in the i^{th} month, the size of which is F_i.

The formula for discounting future cash flow is known, whose present value is denoted by PV and

$$PV = \frac{F_1}{1 + \frac{r}{100}} + \frac{F_2}{\left(1 + \frac{r}{100}\right)^2} + \dots + \frac{F_n}{\left(1 + \frac{r}{100}\right)^n},$$

where r is the minimum average monthly rate of return required by the investor.

The difference $PV - P_0$ gives the net present value of the investment.

$$NPV = \sum_{i=1}^{n} \frac{F_i}{\left(1 + \frac{r}{100}\right)^i} - P_0. \tag{3}$$

It is clear that if $PV \geq P_0$, then $NPV \geq 0$ and this means that the present value of the investment is not less than its cost and this cost is acceptable.

If $NPV < 0$, which is obtained at $PV < P_0$, and indicates that the investment is offered at a higher price than its actual present value. This leads to the conclusion that this investment should not be accepted.

Let's now look at what has been happening in practice recently, in conditions of a global recession, a steady trend of inflation and the remaining low deposit rates in banks. Therefore, taking into account the real interest rates of banks on deposits and the introduced service charges, which in our opinion are significantly high and do not correspond to their low interest rates, we will present a specific example (there are many others) that an investment can also be accepted in case of negative NPV.

Let $K = 20{,}000$ BGN, which means that we are talking about the majority of citizens who can actually have excess funds of the same amount that they have saved so far. On the other hand, we will make a comparison with an investment horizon of one year, i.e. $n = 12$. A normal interest rate at this time (we would say even high for a number of banks) on a term monthly deposit is $\bar{p} = 0{,}12\%$, in our opinion, a complex annual accumulation. At the same time, a common service fee is $t = 2.50$

BGN per month. This, in our opinion, is exactly what the customer of a bank pays for himself, because his money is in a safer place.

It is clear that in the first case under consideration, the nominal amount that is stored in the home will again be BGN 20,000 after one year. The actual present value of this amount will be even less.

Now we will consider the second case in which we deposit the amount of a deposit in a bank.

We use formula (2), such as K = 20000 BGN, n = 12, t = 2,5, $p = \frac{\bar{p}}{12} = \frac{0.12}{12} = 0.01$, then $q = 1 + \frac{0.01}{100} = 0.0001$.

Then

$$K_{12} = 20{,}000.\,1.0001^{12} - 2.5.\frac{1.0001^{12} - 1}{0.0001} = 19{,}994\ BGN$$

which is the amount on the account that we will have at the end of the first year, if it is placed on a term monthly deposit. This shows that regardless of the interest rates and the complicated remuneration in the account we will have 6 BGN less than originally deposited, which is due to the too high fee for servicing the deposit, as we have already said.

By the way, this reduction of our funds has recently imposed on society the concept of “negative interest rate”, i.e. the interest rate on deposits in banks is negative. There is some logic in this, if we look actually at the situation, that part of the interest (but in the opposite direction) is in the fee and so it really turns out that the interest rate on deposits is negative. Let's calculate it in this form. Let the real monthly rate be x. Then:

$$20{,}000\left(1 + \frac{x}{100}\right)^{12} = 19{,}994$$

or

$$\left(1 + \frac{x}{100}\right)^{12} = \frac{19{,}994}{20{,}000},$$

where:

$$1 + \frac{x}{100} = \sqrt[12]{\frac{19{,}994}{20{,}000}}$$

and

$$x = \left(\sqrt[12]{\frac{19{,}994}{20{,}000}} - 1\right).\,100 = -0.0025\%\ per\ month,$$

or

$$12.\,(-0.0025) = -0.03\%\ yearly,$$

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or this is the percentage we lose annually of the deposited amount of BGN 20,000.

It is normal in this situation to look for an opportunity to invest this amount somewhere where we have a monthly return of at least 0.01% (as much as in the bank), but not have other monthly expenses (fees).

It turns out that we have such an opportunity and the proposal is at the end of each month to receive incomes in the amount of BGN 1,667, which means that even if we keep any incomes at home, then at the end of the year we will have 12*1667 = 20,004 BGN, which is 10 BGN more than to keep this amount in the bank. It is true that we mentioned that investments are riskier than a deposit in the bank, but if we ignore this at this case and consider this investment to be reliable (even more so with such a low return), it should be acceptable to us.

Let's see if this is the case if we calculate the NPV for this investment.

First we will further develop formula (3) in this case

$$F_1 = F_2 = \cdots = F_n = F.$$

$$NPV = \sum_{i=1}^{n} \frac{F_i}{\left(1 + \frac{r}{100}\right)^i} - P_0 = F \sum_{i=1}^{n} \frac{1}{q^i} - P_0 = \frac{F}{q^n} \sum_{i=0}^{n-1} q^i - P_0$$

or

$$NPV = \frac{F}{q^n} \frac{q^n - 1}{q - 1} - P_0. \qquad (3')$$

In the specific example:

$$F = 1{,}667, \quad r = 0.01 \ (or \ q = 1.0001),$$
$$n = 12 \ \ and \ \ P_0 = K = 20{,}000.$$

Then from formula (3') follows

$$NPV = \frac{1{,}667}{1.0001^{12}} . \frac{1.0001^{12} - 1}{0.0001} - 20{,}000 = -9 < 0.$$

This example shows that according to the principles of valuation of an investment, proceeding from the value of its *NPV*, this investment should be rejected. Given that other investment opportunities are not available for this amount, it remains either a first case or a second case.

As we have seen from the analysis, the investment with a negative NPV is the best of the three cases considered. Again, we want to emphasize that we do not make a risk assessment here.

After this example examined, one would say that this is a very special case. On the one hand we would answer that it may be a special case, but it shows that one cannot always reject an investment with a negative NPV. On the other hand, we will expand on the example considered.

Let there be no fee for servicing the deposit and the other parameters are the same. In the second case, this means that we will use formula (1) as

$$q = 1.0001, \;\; n = 12 \;\; and \;\; K = 20{,}000.$$

Then

$$K_{12} = 20{,}000.\,1.0001^{12} = 20{,}024 \;\; BGN$$

and the amount will increase by BGN 24 if it is invested in a bank under the conditions mentioned.

Let the client (investor) set a minimum level of return higher than that in the bank, for example r = 0.02% per month (0.02 > 0.01 and it is twice as large). The opportunity he is given is: he invests BGN 20,000 now and takes after a year BGN 20030.

At first sight, it is clear that this is preferable to the deposit in the bank, as at the end of the year there will be BGN 6 more, but let's see what the results would show, after calculating NPV.

In the case of

$$P_0 = 20{,}000, \;\; F_1 = F_2 = \cdots = F_{11} = 0, \;\; F_{12} = 20{,}030,$$

$$q = 1 + \frac{0.02}{100} = 1.0002 \;\; and \;\; n = 12.$$

Then:

$$NPV = \frac{20{,}030}{1.0002^{12}} - 20{,}000 = -18 < 0$$

and we should reject the investment.

Here, of course, the result is such because the minimum level of return required by the investor is higher than the interest rate in the bank, but in the absence of other alternatives, the investment should be accepted, provided that the investor wants a higher average monthly rate of return than the bank offers.

Another problem is what happens to significantly larger amounts of investment. In such cases, under certain conditions it can also be shown that an investment with a negative *NPV* may be preferable.

In the present paper, the statement that not under all conditions a negative *NPV* means a loss-making investment was proved by a specific numerical example. This dependence can be generalized and derived in the general case in a more thorough study. But even this particular example proves that for each specific economic situation it is necessary to make a more thorough quantitative analysis of the profitability of investments, especially if they are of high value.

Conclusions

The article examines three main options for owners of free financial resources, namely to store them at home, to deposit them in a bank or to invest them. Using the methods of financial mathematics, and in particular the formulas for interest and discounting of monetary amounts, it is shown how an amount will change in each of the three variants.

As is known, economic theory and practice recommend making investments in the presence of large amounts of free cash. However, each investment must be accompanied by a quantitative assessment of profitability. Different methods of investment evaluation are known in the literature, each suitable for application under different conditions. The article uses the method of calculating NPV (Net Present Value) based on discounting future cash flows. It is generally accepted that an investment is profitable if there is an NPV greater than zero. A negative NPV indicates that the investment is loss-making and should be rejected. However, this rule has been derived in the presence of many constraints and conditions in the model.

Based on a specific numerical example, this article has shown that it is possible for an investment to be profitable even with a negative NPV. This is obtained by taking specific features of the economic environment, namely a state of recession. It is characterized by low interest rates on deposits in banks and high fees for servicing a bank account. Taking into account these peculiarities, it has been experimentally proven that an investment with a negative NPV is preferable from a deposit in a bank with relatively high monthly account service fees.

The reasoning presented in the paper shows that in any particular situation that needs quantitative analysis, it is necessary to take into account the specific features of the environment and to derive new models and valid statements, since the direct application of commonly known rules can lead to wrong conclusions and losses for economic entities.

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